Pergamon

Int. J. Solids Structures Vol. 32, No. 21, pp. 3129–3143, 1995

0020–7683/95 $9.50 + .00

0020–7683(94)00266–5

ON THE DETERMINATION OF NORMALIZED NONLINEAR MECHANICAL PROPERTIES OF COMPOSITE MATERIALS WITH PERIODICALLY CURVED LAYERS

S. D. AKBAROV
Yıldız Technical University, Faculty of Chemistry and Metallurgy, Department of Metallurgy, 80750 Beşiktaş, Yıldız Istanbul, Turkey

(*Received 7 July 1994; in revised form 21 October 1994*)

Abstract — In several works the normalized mechanical properties of composite materials with curved structures have been studied. These investigations have been carried out relying upon various approximate theories by employing different kinds of hypotheses. Furthermore, there are a few papers in which the above investigations have been carried out using the results obtained within the framework of the piecewise-homogeneous body model with the use of exact three-dimensional equations of linear theory of elasticity. However, under determination of the normalized mechanical properties, these results do not enable one to take into account the nonlinear effects owing to the curvature of reinforcing layers of composite materials. Using a concrete problem as an example, the above method is developed for problems of stress-deformation distribution in such composite materials within a geometrically nonlinear framework and on the base of the obtained results, the nonlinear normalized mechanical properties of composite material with spatially periodically curved layers are determined.

1. INTRODUCTION

In series of investigations, Bazhant (1968), Bolotin and Novichkov (1980), Khoroshun and Maslov (1980), Whitney (1966) and others have studied normalized mechanical properties of composite materials with curved structures. Note that these investigations have been carried out on the basis of various approximate theories using different kinds of hypotheses. In the papers of Akbarov and Guz' (1984, 1985), based on the piecewise-homogeneous body model using exact three-dimensional linear equations of the theory of elasticity, a method through which the stress-deformation state in laminated composites with the curved structures may be obtainable has been proposed. The investigations carried out recently using the method of Akbarov and Guz' (1984) [the review of these investigations is given in detail by Akbarov and Guz' (1992)] enabled Akbarov *et al.* (1992) to determine the normalized mechanical properties of the above composite materials using the results obtained within the framework of the piecewise-homogeneous body model. However, using determination of the normalized mechanical propeties, the results of the research reviewed in the paper by Akbarov and Guz' (1992) do not allow nonlinear effects owing to the curvature of reinforcing layers of composite materials to be taken into account. It is evident that these nonlinear effects may be taken into account only using the results obtained from the exact equations of the nonlinear theory of elasticity. Thus, in the present paper, with a concrete problem as an example, the method by Akbarov and Guz' (1984) is developed for the case of problems of stress-deformation distribution in composite materials with curved structures within a geometrically nonlinear framework. By employing the obtained results, the nonlinear normalized mechanical properties of composite material with spatially periodically curved layers are determined.

2. FORMULATION AND METHOD OF SOLUTION OF STRESS-DEFORMATION STATE PROBLEMS

Consider the laminated composite material which has an infinite number of cophasically curved layers alternating in the direction of the Ox_2 axis (see Fig. 1). Suppose that these layers are periodically curved in the directions of the Ox_1 and Ox_3 axes. The values related to the matrix will be denoted by superscripts (1), and the values related to the filler by superscripts (2). Taking into account the periodicity of the composite structure shown in Fig. 1 in the direction of the Ox_2 axis with period $2(H^{(2)}+H^{(1)})$, where $2H^{(1)}$ is a thickness of the matrix layer and $2H^{(2)}$ is a thickness of the filler layer, among the layers considered we single out two of them, i.e. $1^{(1)}$, $1^{(2)}$ (see Fig. 1), and discuss them below. We associate the corresponding Lagrangian coordinates $O^{(k)}x_1^{(k)}x_2^{(k)}x_3^{(k)}$ $(k = 1,2)$ which in their natural state coincide with Cartesian coordinates and are obtained from $Ox_1x_2x_3$ by parallel transfer along the Ox_2 axis, with the middle surface of each layer of the filler and the matrix.

Throughout the investigations repeated indices are summed over their ranges, however, underlined repeated indices $\underline{k}$, $\underline{i}$ and $(\underline{k})$ are not summed. The angle brackets $\langle\rangle$ are also used throughout the paper to denote the integral throughout the volume of the representative element.

Now we investigate the stress-deformation state in the above body under loading "at infinity" by uniformly distributed normal forces of intensity p_1 in the direction of the Ox_1 axis. Note that p_1 denotes the stress averaged over the total area of the considered body affected by the normal external force in the Ox_1 direction.

Assume that the matrix and filler layer materials are homogeneous, isotropic and linearly elastic. For each layer we write the equilibrium equations, Hooke's law and geometrical relations as follows

$$\frac{\partial}{\partial x_i^{(\underline{k})}}\left[\sigma_{in}^{(\underline{k})}\left(\delta_i^n+\frac{\partial u_i^{(\underline{k})}}{\partial x_n^{(\underline{k})}}\right)\right]=0$$

$$\sigma_{ij}^{(k)}=\lambda^{(\underline{k})}\theta^{(\underline{k})}\delta_i^j+2\mu^{(\underline{k})}\varepsilon_{ij}^{(\underline{k})},\quad \theta^{(k)}=\varepsilon_{11}^{(k)}+\varepsilon_{22}^{(k)}+\varepsilon_{33}^{(k)},$$

$$2\varepsilon_{ij}^{(k)}=\frac{\partial u_i^{(\underline{k})}}{\partial x_j^{(\underline{k})}}+\frac{\partial u_j^{(\underline{k})}}{\partial x_i^{(\underline{k})}}+\frac{\partial u_n^{(\underline{k})}}{\partial x_i^{(\underline{k})}}\frac{\partial u_n^{(\underline{k})}}{\partial x_j^{(\underline{k})}},\quad i,j,n=1,2,3,\quad k=1,2. \tag{1}$$

The notations used in eqn (1) are conventional.

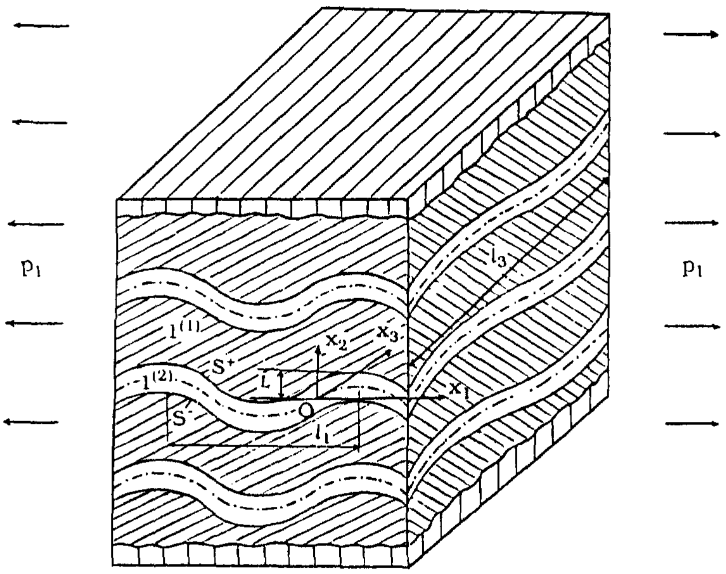

Fig. 1. The structure of composite material with alternating cophasically spatially periodically curved layers.

Suppose that the conditions of complete cohesion are fulfilled at the interface between matrix and filler materials. On the basis of the above, the contact conditions can be written in the following form:

$$\left[\left(\delta_i^n + \frac{\partial u_i^{(1)}}{\partial x_n^{(1)}}\right)\sigma_{jn}^{(1)}\right]\Bigg|_{S^\pm} n_j^\pm = \left[\left(\delta_i^n + \frac{\partial u_i^{(2)}}{\partial x_n^{(2)}}\right)\sigma_{jn}^{(2)}\right]\Bigg|_{S^\pm} n_j^\pm$$

$$u_i^{(1)}|_{S^\pm} = u_i^{(1)}|_{S^\pm}, \tag{2}$$

where S^+ (S^-) is upper (lower) surface of $l^{(2)}$ layer of the filler (see Fig. 1), n_j^+ (n_j^-) are the orthonormal components to the surface S^+ (S^-).

We write the equation of the middle surface of the layer $l^{(2)}$ as

$$x_2^{(2)} = F(x_1^{(2)}, x_3^{(2)}) = \varepsilon f(x_1^{(2)}, x_3^{(2)}). \tag{3}$$

Here $\varepsilon \in [0,a)$ is a dimensionless small parameter and $a < 1$. The geometric meaning of ε and a will be described by the specifically prescribed form of function (3). We assume that the function $F(x_1^{(2)}, x_3^{(2)})$ and its first order derivatives are continuous and satisfy the following condition:

$$\left(\frac{\partial F}{\partial x_1^{(2)}}\right)^2 + \left(\frac{\partial F}{\partial x_3^{(2)}}\right)^2 < 1. \tag{4}$$

In the considered case the function (3) is selected in the form:

$$x_2^{(2)} = L_1 \sin\frac{2\pi}{l_1}x_1^{(2)} \cos\frac{2\pi}{l_3}x_3^{(2)}, \tag{5}$$

where L_1 is the length of the arrow of curving rise, l_1 and l_3 are the wavelength of the forms of middle surface curving in the Ox_1 and Ox_3 directions, respectively.

With the assumption $L_1 < l_1$, the value of the small parameter is selected as

$$\varepsilon = \frac{L_1}{l_1}. \tag{6}$$

Introducing the parameter $\gamma = l_1/l_3$ and taking into account eqns (5) and (6), the above mentioned a is defined from eqn (4) as:

$$a = (2\pi(1+\gamma^2)^{1/2})^{-1}. \tag{7}$$

Thus, by the above, within the framework of the piecewise-homogeneous body model with the use of exact three-dimensional equations of the nonlinear theory of elasticity with only geometrical nonlinearities, the problem is treated exhaustively. For the solution of this problem the following method is proposed.

The values characterizing the stress-deformation state of arbitrary $1^{(k)}$th layer are sought in series form in terms of the parameter ε as follows:

$$\{\sigma_{ij}^{(k)}, \varepsilon_{ij}^{(k)}, u_i^{(k)}\} = \sum_{q=0}^{\infty} \varepsilon^q \{\sigma_{ij}^{(k),q}, \varepsilon_{ij}^{(k),q}, u_i^{(k),q}\}. \tag{8}$$

Using the condition of constant thickness of the filler layers and eqns (3) and (4) the following equations are derived for surfaces $S^\pm$ (see Fig. 1):

$$x_1^{(2)\pm} = t_1 \mp H^{(2)}\varepsilon \frac{\partial f}{\partial t_1} \pm \tfrac{1}{2} H^{(2)}\varepsilon^3 \left[\left(\frac{\partial f}{\partial t_1}\right)^2 + \left(\frac{\partial f}{\partial t_3}\right)^2 \right] \frac{\partial f}{\partial t_1} + \cdots,$$

$$x_2^{(2)\pm} = \mp H^{(2)} + \varepsilon f(t_1, t_3) \mp \tfrac{1}{2} H^{(2)}\varepsilon^2 \left[\left(\frac{\partial f}{\partial t_1}\right)^2 + \left(\frac{\partial f}{\partial t_3}\right)^2 \right] + \cdots,$$

$$x_3^{(2)\pm} = t_3 \mp H^{(2)}\varepsilon \frac{\partial f}{\partial t_3} \pm \tfrac{1}{2} H^{(2)}\varepsilon^3 \left[\left(\frac{\partial f}{\partial t_1}\right)^2 + \left(\frac{\partial f}{\partial t_3}\right)^2 \right] \frac{\partial f}{\partial t_3} + \cdots, \tag{9}$$

where t_1 and t_3 are parameters and $t_1, t_3 \in (-\infty, +\infty)$.

Similar expressions can also be derived for $n_j^{\pm}$. Taking into account the expressions of the $x_i^{(2)\pm}, n_j^{\pm}$ we expand the values of each approximation (8) in series in the vicinity of $(t_1, \pm H^{(k)}, t_3)$ and after some operations we obtain from eqn (2) for each approximation the corresponding contact relations. The contact relations associated with the qth approximation contain the values of all previous approximations. Substituting (8) in (1) and comparing equal powers of ε to describe each approximation, we obtain the corresponding closed system equations. Owing to the linearity of Hooke's law, it will be satisfied for every approximation (8) separately. The remaining relations obtained from eqn (1) for every qth approximation contain the values of all previous approximations.

To simplify the exposition, we write the above relations for the zeroth and first approximations.

The zeroth approximation

In this case eqn (1) holds and we obtain from eqn (2) the following contact conditions:

$$\left[\left(\delta_i^n + \frac{\partial u_i^{(1)}}{\partial x_n^{(1)}}\right)\sigma_{in}^{(1),0}\right]\Bigg|_{(t_1, \mp H^{(1)}, t_3)} = \left[\left(\delta_i^n + \frac{\partial u_i^{(2)}}{\partial x_n^{(2)}}\right)\sigma_{in}^{(2),0}\right]\Bigg|_{(t_1, \pm H^{(2)}, t_3)},$$

$$u_i^{(1),0}\big|_{(t_1, \mp H^{(1)}, t_3)} = u_i^{(2),0}\big|_{(t_1, \pm H^{(2)}, t_3)}. \tag{10}$$

The first approximation

The equilibrium equations in terms of displacements are

$$L_{ij}^{(k)} u_j^{(k),1} = 0, \tag{11}$$

whereas geometrical relations are given by

$$\varepsilon_{ij}^{(k),1} = \tfrac{1}{2}(l + g_i^{(k)}) \frac{\partial u_i^{(k),1}}{\partial x_j^{(k)}} + \tfrac{1}{2}(1 + g_i^{(k)}) \frac{\partial u_i^{(k),1}}{\partial x_j^{(k)}}, \tag{12}$$

and contact conditions become

$$\frac{\partial f}{\partial x_1}\bigg|_{(t_1, t_3)} [(1+g_1^{(1)})\sigma_{11}^{(1),0} - (1+g_1^{(2)})\sigma_{11}^{(2),0}] = (1+g_1^{(2)})\sigma_{12}^{(2),1}\big|_{(t_1, \pm H^{(2)}, t_3)} - (1+g_1^{(1)})\sigma_{12}^{(1),1}\big|_{(t_1, \mp H^{(1)}, t_3)},$$

$$(1+g_2^{(1)})\sigma_{22}^{(1),1}\big|_{(t_1, \mp H^{(1)}, t_3)} = (1+g_2^{(2)})\sigma_{22}^{(2),1}\big|_{(t_1, \pm H^{(2)}, t_3)},$$

$$-\frac{\partial f}{\partial x_3}\bigg|_{(t_1, t_3)} [(1+g_3^{(1)})\sigma_{33}^{(1),0} - (1+g_3^{(2)})\sigma_{33}^{(2),0}] = (1+g_3^{(2)})\sigma_{23}^{(2),1}\big|_{(t_1, \pm H^{(2)}, t_3)} - (1+g_3^{(1)})\sigma_{23}^{(1),1}\big|_{(t_1, \mp H^{(1)}, t_3)},$$

$$u_i^{(1),1}\big|_{(t_1, \mp H^{(1)}, t_3)} = u_i^{(2),1}\big|_{(t_1, \pm H^{(2)}, t_3)}. \tag{13}$$

In eqn (11), $L_{ij}^{(k)}$ are operators and they are defined by the following expressions:

$$L_{11}^{(k)} = \sum_{i=1}^{3} C_{i1}^{(k)} \frac{\partial^2}{\partial (x_i^{(k)})^2} \quad L_{12}^{(k)} = C_{14}^{(k)} \frac{\partial^2}{\partial x_1^{(k)} \partial x_2^{(k)}} \quad L_{13}^{(k)} = C_{15}^{(k)} \frac{\partial^2}{\partial x_1^{(k)} \partial x_3^{(k)}}$$

$$L_{21}^{(k)} = C_{24}^{(k)} \frac{\partial^2}{\partial x_1^{(k)} \partial x_2^{(k)}} \quad L_{22}^{(k)} = \sum_{i=1}^{3} C_{2i}^{(k)} \frac{\partial^2}{\partial (x_i^{(k)})^2} \quad L_{23}^{(k)} = C_{25}^{(k)} \frac{\partial^2}{\partial x_2^{(k)} \partial x_3^{(k)}}$$

$$L_{31}^{(k)} = C_{34}^{(k)} \frac{\partial^2}{\partial x_1^{(k)} \partial x_3^{(k)}} \quad L_{32}^{(k)} = C_{35}^{(k)} \frac{\partial^2}{\partial x_2^{(k)} \partial x_3^{(k)}} \quad L_{33}^{(k)} = \sum_{i=1}^{3} C_{3i}^{(k)} \frac{\partial^2}{\partial (x_i^{(k)})^2}. \tag{14}$$

The following notations are used in (14):

$$C_{11}^{(k)} = (1+g_1^{(k)})^2(1+\Lambda^{(k)}) + \frac{\sigma_{11}^{(k),0}}{E^{(k)}}(1+\nu^{(k)}), \quad C_{12}^{(k)} = \tfrac{1}{2}(1+g_1^{(k)})^2,$$

$$C_{13}^{(k)} = \tfrac{1}{2}(1+g_1^{(k)})^2 + \frac{\sigma_{33}^{(k),0}}{E^{(k)}}(1+\nu^{(k)}), \quad C_{14}^{(k)} = (\tfrac{1}{2}+\Lambda^{(k)})(1+g_1^{(k)})(1+g_2^{(k)}),$$

$$C_{15}^{(k)} = (\tfrac{1}{2}+\Lambda^{(k)})(1+g_1^{(k)})(1+g_3^{(k)}), \quad C_{21}^{(k)} = \tfrac{1}{2}(1+g_2^{(k)})^2 + \frac{\sigma_{11}^{(k),0}}{E^{(k)}}(1+\nu^{(k)}),$$

$$C_{22}^{(k)} = (1+\Lambda^{(k)})(1+g_2^{(k)})^2, \quad C_{23}^{(k)} = \tfrac{1}{2}(1+g_2^{(k)})^2 + \frac{\sigma_{33}^{(k),0}}{E^{(k)}}(1+\nu^{(k)}),$$

$$C_{24}^{(k)} = (\tfrac{1}{2}+\Lambda^{(k)})(1+g_1^{(k)})(1+g_2^{(k)}), \quad C_{25}^{(k)} = (\tfrac{1}{2}+\Lambda^{(k)})(1+g_2^{(k)})(1+g_3^{(k)}),$$

$$C_{31}^{(k)} = \tfrac{1}{2}(1+g_3^{(k)})^2 + \frac{\sigma_{11}^{(k),0}}{E^{(k)}}(1+\nu^{(k)}), \quad C_{32}^{(k)} = \tfrac{1}{2}(1+g_3^{(k)})^2,$$

$$C_{33}^{(k)} = (1+g_3^{(k)})^2(1+\Lambda^{(k)}) + \frac{\sigma_{33}^{(k),0}}{E^{(k)}}(1+\nu^{(k)}),$$

$$C_{34}^{(k)} = (\tfrac{1}{2}+\Lambda^{(k)})(1+g_1^{(k)})(1+g_3^{(k)}), \quad C_{35}^{(k)} = (\tfrac{1}{2}+\Lambda^{(k)})(1+g_2^{(k)})(1+g_3^{(k)}), \tag{15}$$

where

$$g_i^{(k)} = \frac{\partial u_i^{(k),0}}{\partial x_i^{(k)}}, \quad \Lambda^{(k)} = \frac{\nu^{(k)}}{1-2\nu^{(k)}}. \tag{16}$$

In eqns (15) and (16), $E^{(k)}$ and $\nu^{(k)}$ are Young's modulus and the Poisson coefficient, respectively.

Note that when the expressions (11)–(16) are written, we take into account that the values of the zeroth approximation correspond to the uniform stress state in every considered layer. Furthermore, note that the analogous equations can be written for the values of the subsequent approximations.

Consider now the determination of the zeroth and first approximations. It follows from the foregoing and from eqn (10) that the zeroth approximation corresponds to the stress-deformation state in a laminated composite in the case of an ideal (uncurved) layout of the layers with a prescribed form of the external forces. Therefore, in the case under consideration for determining the zeroth order approximation, the nonlinear terms in eqns (1) and (10) can be neglected with sufficiently high accuracy. Thus, the zeroth approximation may be assumed to correspond to geometrical linearity. In this case, from the statement of the problem on the zeroth approximation, we may write

$$\sigma_{12}^{(k),0} = \sigma_{22}^{(k),0} = \sigma_{13}^{(k),0} = \sigma_{23}^{(k),0} = 0, \quad k = 1,2. \tag{17}$$

Taking into account eqns (1) and (17), and the relations

$$\eta^{(1)}\sigma_{11}^{(1),0} + \eta^{(2)}\sigma_{11}^{(2),0} = p_1, \quad \eta^{(1)}\sigma_{33}^{(1),0} + \eta^{(2)}\sigma_{33}^{(2),0} = 0, \tag{18}$$

we obtain the following expressions for the zeroth approximation:

$$\sigma_{11}^{(2),0} = p_1\left(\eta^{(1)}\frac{E^{(1)}}{E^{(2)}} + \eta^{(2)} - \nu^{(1)}\left(\nu^{(1)}\eta^{(2)} + \nu^{(2)}\eta^{(1)}\frac{E^{(1)}}{E^{(2)}}\right)\right)$$
$$\times\left(\left(\eta^{(1)}\frac{E^{(1)}}{E^{(2)}} + \eta^{(2)}\right)^2 - \left(\nu^{(1)}\eta^{(2)} + \nu^{(2)}\eta^{(1)}\frac{E^{(1)}}{E^{(2)}}\right)^2\right)^{-1},$$

$$\sigma_{33}^{(2),0} = -\frac{\nu^{(1)}p_1}{\eta^{(1)}\dfrac{E^{(1)}}{E^{(2)}} + \eta^{(2)}} + \frac{\nu^{(1)}\eta^{(2)} + \nu^{(2)}\eta^{(1)}\dfrac{E^{(1)}}{E^{(2)}}}{\eta^{(1)}\dfrac{E^{(1)}}{E^{(2)}} + \eta^{(2)}}\sigma_{11}^{(2),0},$$

$$\sigma_{11}^{(1),0} = \frac{p_1}{\eta^{(1)}} - \frac{\eta^{(2)}}{\eta^{(1)}}\sigma_{11}^{(2),0}, \quad \sigma_{33}^{(1),0} = -\frac{\eta^{(2)}}{\eta^{(1)}}\sigma_{33}^{(2),0},$$

$$u_1^{(k),0} = \frac{1}{E^{(k)}}(\sigma_{11}^{(k),0} - \nu^{(k)}\sigma_{33}^{(k),0})x_1^{(k)}, \quad u_2^{(k),0} = -\frac{\nu^{(k)}}{E^{(k)}}(\sigma_{11}^{(k),0} + \sigma_{33}^{(k),0})x_2^{(k)} + c^{(k)},$$

$$u_3^{(k),0} = \frac{1}{E^{(k)}}(\sigma_{33}^{(k),0} - \nu^{(k)}\sigma_{11}^{(k),0})x_3^{(k)}, \quad \eta^{(k)} = \frac{H^{(k)}}{H^{(1)} + H^{(2)}}, \quad c^{(k)} = \text{const.} \tag{19}$$

Note that in the case, when $\nu^{(1)} = \nu^{(2)}$, we obtain $\sigma_{33}^{(1),0} = \sigma_{33}^{(2),0} = 0$ from eqn (19).

Consider now the determination of the first order approximation. In this case, we suppose that $\nu^{(1)} = \nu^{(2)} = \nu$. Taking into account eqns (5) and (13), the displacements in this approximation are represented in the following form:

$$u_i^{(k),1} = \varphi_i^{(k)}(x_2^{(k)})\left(\cos\frac{2\pi}{l_1}x_1^{(k)}\cos\frac{2\pi}{l_3}x_3^{(k)}\delta_i^1 + \sin\frac{2\pi}{l_1}x_1^{(k)}\cos\frac{2\pi}{l_3}x_3^{(k)}\delta_i^2 + \sin\frac{2\pi}{l_1}x_1^{(k)}\sin\frac{2\pi}{l_3}x_3^{(k)}\delta_i^3\right). \tag{20}$$

Substituting eqn (20) into (11), we obtain the following system of ordinary differential equations for the determination of the unknown functions $\varphi_i^{(k)}$:

$$\frac{d^2\varphi_i^{(k)}}{d(x_2^{(k)})^2} + \alpha_{11}^{(k)}\varphi_i^{(k)} + \delta_i^1\left(\alpha_{12}^{(k)}\frac{d\varphi_2^{(k)}}{dx_2^{(k)}} + \alpha_{13}^{(k)}\varphi_3^{(k)}\right)$$
$$+\delta_i^2\left(\alpha_{22}^{(k)}\frac{d\varphi_1^{(k)}}{dx_2^{(k)}} + \alpha_{23}^{(k)}\frac{d\varphi_3^{(k)}}{dx_2^{(k)}}\right) + \delta_i^3\left(\alpha_{32}^{(k)}\varphi_1^{(k)} + \alpha_{33}^{(k)}\frac{d\varphi_2^{(k)}}{dx_2^{(k)}}\right) = 0, \quad 0 = 1,2,3, \tag{21}$$

where

$$\alpha_{11}^{(k)} = -\frac{C_{11}^{(k)} + C_{13}^{(k)}\gamma^2}{C_{12}^{(k)}}, \quad \alpha_{12}^{(k)} = \frac{C_{14}^{(k)}}{C_{12}^{(k)}}, \quad \alpha_{13}^{(k)} = \frac{C_{15}^{(k)}}{C_{12}^{(k)}}\gamma,$$

$$\alpha_{21}^{(k)} = -\frac{C_{21}^{(k)} + C_{23}^{(k)}\gamma^2}{C_{22}^{(k)}}, \quad \alpha_{22}^{(k)} = -\frac{C_{24}^{(k)}}{C_{22}^{(k)}}, \quad \alpha_{23}^{(k)} = \frac{C_{25}^{(k)}}{C_{22}^{(k)}}\gamma,$$

$$\alpha_{31}^{(k)} = -\frac{C_{31}^{(k)} + C_{33}^{(k)}\gamma^2}{C_{32}^{(k)}}, \quad \alpha_{32}^{(k)} = \frac{C_{34}^{(k)}}{C_{32}^{(k)}}, \quad \alpha_{33}^{(k)} = -\frac{C_{31}^{(k)}}{C_{32}^{(k)}}\gamma. \tag{22}$$

Following some transformations and taking into account eqns (10)–(22), the representations for the functions $\varphi_i^{(k)}$ are obtained as:

$$\varphi_1^{(k)}(x_2^{(k)}) = \left[\frac{d^2}{d(x_2^{(k)})^2} - \gamma^2 - \frac{1-2\nu}{1-\nu}\frac{\lambda(1+\nu)}{(1-\nu\lambda)^2} - \frac{1-2\nu}{2(1-\nu)}\right]\chi^{(k)},$$

$$\varphi_2^{(k)}(x_2^{(k)}) = -\alpha_{22}^{(k)}\frac{d}{dx_2^{(k)}}\chi^{(k)} + \left[\left(\frac{\alpha_{13}^{(k)}\alpha_{33}^{(k)}}{\alpha_{12}^{(k)}} + \alpha_{11}^{(k)}\right) + \left(\frac{\alpha_{13}^{(k)}\alpha_{33}^{(k)}}{\alpha_{12}^{(k)}} - \alpha_{31}^{(k)}\right)(\alpha_{11}^{(k)}\alpha_{31}^{(k)} - \alpha_{13}^{(k)}\alpha_{32}^{(k)})\right]\Psi^{(k)},$$

$$\varphi_3^{(k)}(x_2^{(k)}) = (\alpha_{22}^{(k)}\alpha_{33}^{(k)} - \alpha_{32}^{(k)2})\chi^{(k)} - \left[\alpha_{33}^{(k)}\left(\frac{\alpha_{13}^{(k)}\alpha_{33}^{(k)}}{\alpha_{12}^{(k)}} - \alpha_{31}^{(k)}\right) + \alpha_{11}^{(k)}\alpha_{33}^{(k)} - \alpha_{12}^{(k)}\alpha_{32}^{(k)}\right]\frac{d}{dx_2^{(k)}}\Psi^{(k)}, \tag{23}$$

where

$$\lambda = \frac{\sigma_{11}^{(1),0}}{E^{(1)}} = \frac{\sigma_{11}^{(2),0}}{E^{(2)}} = \varepsilon_{11}^{(1),0} = \varepsilon_{11}^{(2),0} \tag{24}$$

and $\chi^{(k)}$, $\Psi^{(k)}$ are the solutions of the following equations:

$$\left(\frac{d^2}{d(x_2^{(k)})^2} - a_0^{(k)}\right)\Psi^{(k)} = 0, \tag{25}$$

$$\left(\frac{d^4}{d(x_2^{(k)})^4} + a_1^{(k)}\frac{d^2}{d(x_2^{(k)})^2} + a_2^{(k)}\right)\chi^{(k)} = 0. \tag{26}$$

In eqns (25) and (26), $a_0^{(k)}, a_1^{(k)}$ and $a_2^{(k)}$ are

$$a_0^{(k)} = \frac{\alpha_{13}^{(k)}\alpha_{33}^{(k)}}{\alpha_{12}^{(k)}} - \alpha_{31}^{(k)}, \quad a_1^{(k)} = -\alpha_{12}^{(k)}\alpha_{22}^{(k)} + \alpha_{11}^{(k)} + \alpha_{31}^{(k)} - \frac{\alpha_{23}^{(k)}\alpha_{12}^{(k)}\alpha_{31}^{(k)}}{\alpha_{13}^{(k)}},$$

$$a_2^{(k)} = -\alpha_{13}^{(k)}\alpha_{32}^{(k)} + \alpha_{31}^{(k)}\alpha_{11}^{(k)} + \alpha_{23}^{(k)}\alpha_{12}^{(k)}\alpha_{32}^{(k)} - \frac{\alpha_{23}^{(k)}\alpha_{12}^{(k)}\alpha_{31}^{(k)}\alpha_{11}^{(k)}}{\alpha_{13}^{(k)}}. \tag{27}$$

Taking into account that the unequality $\lambda \ll 1$ is practically true, we prove by direct verification that the roots of characteristic equations corresponding to the differential equations (25) and (26) are real and complex numbers, respectively. Furthermore, taking into account that in the considered case, $\sigma_{12}^{(k),1}$ and $\sigma_{23}^{(k),1}$ must be even functions with respect to $x_2^{(k)}$, the solutions of eqns (25) and (26) are selected in the following form:

$$\Psi^{(k)} = A_1^{(k)} \cosh\left(\sqrt{a_0^{(k)}} 2\pi \frac{x_2^{(k)}}{l_1}\right),$$

$$\chi^{(k)} = B_1^{(k)} \cosh\left(\lambda_1^{(k)} 2\pi \frac{x_2^{(k)}}{l_1}\right) \sin\left(\lambda_2^{(k)} 2\pi \frac{x_2^{(k)}}{l_1}\right)$$

$$+ B_2^{(k)} \sinh\left(\lambda_1^{(k)} 2\pi \frac{x_2^{(k)}}{l_1}\right) \cos\left(\lambda_2^{(k)} 2\pi \frac{x_2^{(k)}}{l_1}\right), \tag{28}$$

where

$$\lambda_{1,2}^{(k)} = [(\lambda_{11}^{(k)})^2 + (\lambda_{21}^{(k)})^2]^{1/4} \left[\frac{1}{2} \pm \frac{\lambda_{11}^{(k)}}{2((\lambda_{11}^{(k)})^2 + (\lambda_{21}^{(k)})^2)^{1/2}}\right]^{1/2}$$

$$\lambda_{11}^{(k)} = -\frac{a_1^{(k)}}{2}, \quad \lambda_{21}^{(k)} = \tfrac{1}{2}(4a_2^{(k)} - (a_1^{(k)})^2)^{1/2}. \tag{29}$$

Using eqns (23) and (1), the equations for the determination of the unknown constants $A_1^{(k)}, B_1^{(k)}, B_2^{(k)}$ entering into eqn (28) are obtained from eqn (9).

Thus from eqns (28), (29) and (23) and from Hooke's law, we completely determine the first order approximation. In this case, the components of the stress and deformation tensors can be represented by

$$\begin{Bmatrix} \sigma_{ij}^{(k),1} \\ \varepsilon_{ij}^{(k),1} \end{Bmatrix} = \begin{Bmatrix} g_{ij}^{(k),1}(x_2^{(k)}) \\ \mathrm{d}_{ij}^{(k),1}(x_2^{(k)}) \end{Bmatrix} \left(\delta_i^j \sin\left(2\pi\frac{x_1}{l_1}\right)\cos\left(2\pi\frac{x_3}{l_3}\right) + \delta_i^1\delta_j^3 \cos\left(2\pi\frac{x_1}{l_1}\right)\sin\left(2\pi\frac{x_3}{l_3}\right)\right.$$

$$\left. + \delta_i^1\delta_j^2 \cos\left(2\pi\frac{x_1}{l_1}\right)\cos\left(2\pi\frac{x_3}{l_3}\right) + \delta_i^2\delta_j^3 \sin\left(2\pi\frac{x_1}{l_1}\right)\sin\left(2\pi\frac{x_3}{l_3}\right)\right), \tag{30}$$

where δ_i^j are Kronecker symbols, $g_{ij}^{(k),1}(x_2^{(k)})$, $d_{ij}^{(k),1}(x_2^{(k)})$ are the known functions defined by eqns (28), (29), (23) and (1). Owing to the cumbersome forms of these functions we prefer not to show them here.

Taking into account some obvious changes and continuing this process in a similar manner, we can also determine, in principle, subsequent orders of approximations, the expressions of which are obtained in the form of the separation of the arguments as in eqn (30).

Thus, from the above the stress-deformation state in the every component of the considered composite material is determined with sufficiently high accuracy.

3. DETERMINATION OF NORMALIZED MECHANICAL PROPERTIES

As is well known, for the determination of the values of the normalized modulus of elasticity, it is necessary first to select the representative element and to define the averaged values of the components of the stress and strain tensors over the volume V of this element, i.e.

$$\langle \sigma_{ij} \rangle = \frac{1}{V}\int_V \sigma_{ij}\,\mathrm{d}V, \quad \langle \varepsilon_{ij} \rangle = \frac{1}{V}\int_V \varepsilon_{ij}\,\mathrm{d}V. \tag{31}$$

However, for definition (31) it is necessary to know the distribution of the stresses and the strains in every component of the representative element.

Consider the calculation of the integrals (31) in the above case. We attempt to present another point of view on the method described in Section 2. We denote by $\tilde{D}^{(k)}$ the region occupied by the $1^{(k)}$th layer in the considered composite. The coordinates of points in the region are denoted by $\tilde{x}_i^{(k)}$ in the corresponding system of coordinates $O^{(k)}x_1^{(k)}x_2^{(k)}x_3^{(k)}$. Thus, the functions described the stress-deformation state in the $1^{(k)}$th layer will be dependent on the coordinates $\tilde{x}_i^{(k)}$.

Now for each region $\tilde{D}^{(k)}$ we choose a strip $D^{(k)}$, the thickness of which is equal to the thickness of the layer occupied by the region $\tilde{D}^{(k)}$. Coordinates of the points of the strip $D^{(k)}$ are denoted by $x_1^{(k)}, x_2^{(k)}$, and $x_3^{(k)}$ where $x_1^{(k)}, x_3^{(k)} \in (-\infty, +\infty)$ and $-H^{(k)} \leqslant x_2^{(k)} \leqslant +H^{(k)}$. It is required that between coordinates $\tilde{x}_i^{(k)}$ and $x_i^{(k)}$ the following relations should be satisfied:

for the matrix layer:

$$\tilde{x}_1^{(1)} = x_1^{(1)} + \varepsilon x_2^{(1)} \frac{H^{(2)}}{H^{(1)}} \frac{\partial f}{\partial x_1} - \varepsilon^3 x_2^{(1)} \frac{H^{(2)}}{2H^{(1)}} \frac{\partial f}{\partial x_1} \left[\left(\frac{\partial f}{\partial x_1} \right)^2 + \left(\frac{\partial f}{\partial x_3} \right)^2 \right] + \cdots,$$

$$\tilde{x}_2^{(1)} = x_2^{(1)} + \varepsilon f(x_1, x_3) + \varepsilon^2 x_2^{(1)} \frac{H^{(2)}}{2H^{(1)}} \left[\left(\frac{\partial f}{\partial x_1} \right)^2 + \left(\frac{\partial f}{\partial x_3} \right)^2 \right] + \cdots,$$

$$\tilde{x}_3^{(1)} = x_3^{(1)} + \varepsilon x_2^{(1)} \frac{H^{(2)}}{H^{(1)}} \frac{\partial f}{\partial x_3} - \varepsilon^3 x_2^{(1)} \frac{H^{(2)}}{2H^{(1)}} \frac{\partial f}{\partial x_3} \left[\left(\frac{\partial f}{\partial x_1} \right)^2 + \left(\frac{\partial f}{\partial x_3} \right)^2 \right] + \cdots. \quad (32)$$

for the filler layer:

$$\tilde{x}_1^{(2)} = x_1^{(2)} - \varepsilon x_2^{(2)} \frac{\partial f}{\partial x_1} + \tfrac{1}{2}\varepsilon^3 x_2^{(2)} \frac{\partial f}{\partial x_1} \left[\left(\frac{\partial f}{\partial x_1} \right)^2 + \left(\frac{\partial f}{\partial x_3} \right)^2 \right] + \cdots,$$

$$\tilde{x}_2^{(2)} = x_2^{(2)} + \varepsilon f(x_1, x_3) - \tfrac{1}{2}\varepsilon^2 x_2^{(2)} \left[\left(\frac{\partial f}{\partial x_1} \right)^2 + \left(\frac{\partial f}{\partial x_3} \right)^2 \right] + \cdots,$$

$$\tilde{x}_3^{(2)} = x_3^{(2)} - \varepsilon x_2^{(2)} \frac{\partial f}{\partial x_3} + \tfrac{1}{2}\varepsilon^3 x_2^{(2)} \frac{\partial f}{\partial x_3} \left[\left(\frac{\partial f}{\partial x_1} \right)^2 + \left(\frac{\partial f}{\partial x_3} \right)^2 \right] + \cdots. \quad (33)$$

In eqns (32) and (33) suppose that $x_1^{(1)} = x_1^{(2)} = x_1$, $x_3^{(1)} = x_3^{(2)} = x_3$. Note that formulae (32) and (33) are obtained by direct verification. The conclusion may be drawn that by using (32) and (33) the points of the strip $D^{(k)}$ are uniquely mapped into points of the region (the layer) $\tilde{D}^{(k)}$.

Taking into account the considerations presented, the essence of the method described in Section 2 may be formulated in the following form. The solutions of the nonlinear equations (1) in the regions $\tilde{D}^{(k)}$ satisfying the contact conditions (2) and the corresponding boundary conditions are reduced to the solutions of the successive linear boundary problems in the regions (strips) $D^{(k)}$. In this case the functions describing the stress-deformation state in layers (regions) $\tilde{D}^{(k)}$ are determined by the following formulae with functions which are the solutions of this series of linear problems for regions $D^{(k)}$:

$$\begin{Bmatrix} \tilde{\sigma}_{ij}^{(k)} \\ \tilde{\varepsilon}_{ij}^{(k)} \end{Bmatrix} (\tilde{x}_1^{(k)}, \tilde{x}_2^{(k)}, \tilde{x}_3^{(k)}) = \sum_{q=0}^{\prime} \varepsilon^q \begin{Bmatrix} P_{ij}^{(k),q} \\ e_{ij}^{(k),q} \end{Bmatrix} (\tilde{x}_1^{(k)}, \tilde{x}_2^{(k)}, \tilde{x}_3^{(k)}), \quad (34)$$

where

$$P_{ij}^{(k),0} = \sigma_{ij}^{(k),0}, \quad P_{ij}^{(k),1} = \sigma_{ij}^{(k),1},$$

$$P_{ij}^{(1),2} = \sigma_{ij}^{(1),2} + x_2^{(1)} \frac{H^{(2)}}{H^{(1)}} \frac{\partial f}{\partial x_1} \frac{\partial \sigma_{ij}^{(1),1}}{\partial x_1} + f(x_1, x_3) \frac{\partial \sigma_{ij}^{(1),1}}{\partial x_2^{(1)}} + x_2^{(1)} \frac{H^{(2)}}{H^{(1)}} \frac{\partial f}{\partial x_3} \frac{\partial \sigma_{ij}^{(1),1}}{\partial x_3},$$

$$P_{ij}^{(2),2} = \sigma_{ij}^{(2),2} - x_2^{(2)} \frac{\partial f}{\partial x_1} \frac{\partial \sigma_{ij}^{(2),1}}{\partial x_1} + f(x_1, x_3) \frac{\partial \sigma_{ij}^{(2),1}}{\partial x_2^{(2)}} - x_2^{(2)} \frac{\partial f}{\partial x_3} \frac{\partial \sigma_{ij}^{(2),1}}{\partial x_3}, \ldots \quad (35)$$

If we substitute $\sigma_{ij}^{(k),q}$ by $\varepsilon_{ij}^{(k),q}$ in eqn (35), we obtain the formulae for the definition of $e_{ij}^{(k),q}$. Note that by writing $(\tilde{x}_1^{(k)}, \tilde{x}_2^{(k)}, \tilde{x}_3^{(k)})$, as represented by eqns (32) and (33), instead of $(x_1^{(k)}, x_2^{(k)}, x_3^{(k)})$ in eqn (8), and expanding the values of every approximation in the series in the vicinity of $(x_1^{(k)}, x_2^{(k)}, x_3^{(k)})$, followed by classifying in equal powers of ε, we can obtain expressions (35). After these preliminary operations, consider the calculation of the integrals (31). For this purpose, we separate the representative element in considered composite. By direct verification we prove that such element may be only the element indicated in Fig. 2. The volume of this element we denote by $\tilde{V}$ and represent it in the form $\tilde{V} = \tilde{V}^{(1)} + \tilde{V}^{(2)}$, where $\tilde{V}^{(2)}(\tilde{V}^{(1)})$ is the part of $\tilde{V}$ filled by the layer of the filler (the matrix). Furthermore, we require that the coordinates of points of the $\tilde{V}$ are determined with the help of eqns (32) and (33) in the case when

$$\frac{l_1}{4} \leqslant x_1^{(k)} \leqslant \frac{3l_1}{4}, \quad 0 \leqslant x_3^{(k)} \leqslant \frac{l_3}{2},$$
$$-H^{(1)} \leqslant x_2^{(1)} \leqslant 0, \qquad 0 \leqslant x_2^{(2)} \leqslant H^{(2)}. \tag{36}$$

After separations of the above representative element, we determine the average values of the stress and strains over the volume $\tilde{V}$ of this element, i.e.

$$\langle \sigma_{ij} \rangle = \eta^{(1)} \langle \tilde{\sigma}_{ij}^{(1)} \rangle + \eta^{(2)} \langle \tilde{\sigma}_{ij}^{(2)} \rangle, \quad \langle \varepsilon_{ij} \rangle = \eta^{(1)} \langle \tilde{\varepsilon}_{ij}^{(1)} \rangle + \eta^{(2)} \langle \tilde{\varepsilon}_{ij}^{(2)} \rangle,$$
$$\left\{ \begin{matrix} \langle \tilde{\sigma}_{ij}^{(k)} \rangle \\ \langle \tilde{\varepsilon}_{ij}^{(k)} \rangle \end{matrix} \right\} = \frac{1}{\tilde{V}^{(\underline{k})}} \int_{\tilde{V}^{(k)}} \left\{ \begin{matrix} \tilde{\sigma}_{ij}^{(\underline{k})} \\ \tilde{\varepsilon}_{ij}^{(\underline{k})} \end{matrix} \right\} (\tilde{x}_1^{(\underline{k})}, \tilde{x}_2^{(\underline{k})}, \tilde{x}_3^{(\underline{k})}) \, \mathrm{d}\tilde{V}^{(\underline{k})}. \tag{37}$$

Taking into account that with the change of the coordinates $\tilde{x}_i^{(k)}$ in the volume $\tilde{V}^{(k)}$ the coordinates $x_i^{(k)}$ in eqn (33) also change, we obtain from eqn (34)

$$\left\{ \begin{matrix} \langle \tilde{\sigma}_{ij}^{(k)} \rangle \\ \langle \tilde{\varepsilon}_{ij}^{(k)} \rangle \end{matrix} \right\} = \sum_{q=0}^{\infty} \varepsilon^q \left\{ \begin{matrix} \langle P_{ij}^{(k),q} \rangle \\ \langle e_{ij}^{(\underline{k}),q} \rangle \end{matrix} \right\} (x_1^{(\underline{k})}, x_2^{(\underline{k})}, x_3^{(\underline{k})}), \tag{38}$$

where

$$\left\{ \begin{matrix} \langle P_{ij}^{(k),q} \rangle \\ \langle e_{ij}^{(k),q} \rangle \end{matrix} \right\} = \frac{4}{H^{(\underline{k})} l_1 l_3} \int_{l_1/4}^{3l_1/4} \left[\int_{-H^{(1)}\delta_1^k}^{H^{(2)}\delta_2^k} \left[\int_0^{l_3/2} \left\{ \begin{matrix} P_{ij}^{(\underline{k}),q} \\ e_{ij}^{(\underline{k}),q} \end{matrix} \right\} (x_1^{(\underline{k})}, x_2^{(\underline{k})}, x_3^{(\underline{k})}) \, \mathrm{d}x_3^{(\underline{k})} \right] \mathrm{d}x_2^{(\underline{k})} \right] \mathrm{d}x_1^{(\underline{k})},$$
$$k = 1, 2. \tag{39}$$

Writing the expressions of $P_{ij}^{(k),q}, e_{ij}^{(k),q}$ by $\sigma_{ij}^{(k),q}, \varepsilon_{ij}^{(k),q}$ and calculating the integrals (39), we

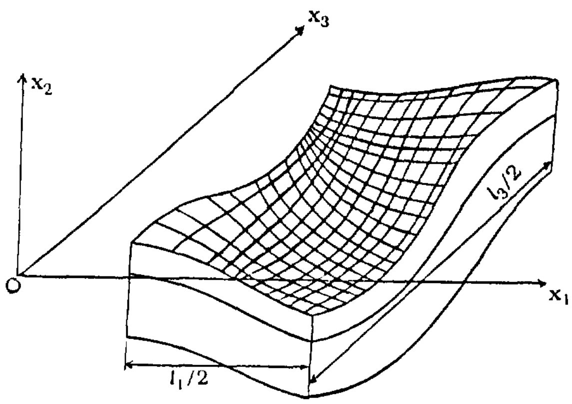

Fig. 2. Representative element selected from the considered composite material.

determine $\langle\sigma_{ij}\rangle$, $\langle\varepsilon_{ij}\rangle$ from eqns (38) and (37). After these operations we define the deformation energy U accumulated in a selected representative element. In the considered case, after some transformations, we obtain

$$U = \frac{1}{2}E_1(1+\varepsilon^2\gamma_2+\varepsilon^4\gamma_4+\dots)\lambda^2, \tag{40}$$

where

$$\lambda = \varepsilon_{11}^{(1),0} = \varepsilon_{11}^{(2),0}, \quad E_1 = \eta^{(1)}E^{(1)}+\eta^{(2)}E^{(2)},$$
$$\gamma_{2k} = \gamma_{2k}(\eta^{(2)}, \kappa, E^{(2)}/E^{(1)}, \gamma, \nu, \lambda), \quad \kappa = 2\pi H^{(2)}/l_1. \tag{41}$$

As an example, we write expressions for γ_2:

$$\gamma_2 = \langle\bar{P}_{11}^2\rangle + \frac{E_1}{E^{(2)}}\langle\bar{e}_{11}^2\rangle + \frac{E_1}{E^{(2)}}\bar{\varepsilon}_{33}^0\langle\bar{P}_{33}^2\rangle + \frac{E_1}{E^{(2)}}\bar{\varepsilon}_{33}^0\langle\bar{P}_{22}^2\rangle + \frac{E_1}{E^{(2)}}\langle\bar{P}_{13}^1\rangle\langle\bar{e}_{13}^1\rangle, \tag{42}$$

where

$$\langle\bar{P}_{ij}^q\rangle = \lambda^{-1}\langle P_{ij}^q\rangle, \quad \langle\bar{e}_{ij}^q\rangle = \lambda^{-1}\langle e_{ij}^q\rangle,$$
$$\begin{Bmatrix}\langle P_{ij}^q\rangle \\ \langle e_{ij}^q\rangle\end{Bmatrix} = \eta^{(1)}\begin{Bmatrix}\langle P_{ij}^{(1),q}\rangle \\ \langle e_{ij}^{(1),q}\rangle\end{Bmatrix} + \eta^{(2)}\begin{Bmatrix}\langle P_{ij}^{(2),q}\rangle \\ \langle e_{ij}^{(2),q}\rangle\end{Bmatrix}. \tag{43}$$

According to the well known procedure, by differentiating the expression U (40) with respect to λ, we define

$$\langle\sigma_{11}\rangle = \frac{\partial U}{\partial\lambda} = \tilde{E}_1(\lambda)\lambda. \tag{44}$$

Thus, we obtain the expression for the construction of the nonlinear dependences between the averaged values of the stress and strain under uniaxial loading in the direction of the Ox_1 axis of the considered composite material having the structure illustrated in Fig. 1. Note that the above nonlinearity arises as a result of the curvature of the reinforcing layers in the composite material. Moreover, note that in eqns (40)–(42), E_1 is a normalized modulus of the considered composite material in the direction of Ox_1 axis (see Fig. 1) in the case of ideal layout of the layers in it. In other words, if $\varepsilon = 0$ we obtain $\tilde{E}_1 = E_1$ from eqn (40).

4. NUMERICAL RESULTS

Now consider a series of numerical results obtained within the framework of the above approach and investigate the influence of the change of structural parameters of composite materials shown in Fig. 1, on the character of relations between $\tilde{E}_1(\lambda)/E_1$ and λ, and between $\langle\sigma_{11}\rangle/E_1$ and λ as well. In this case, we use only the first two terms of the series (40) and suppose that $\nu^{(1)} = \nu^{(2)} = 0.3$, $\varepsilon = 0.025$ (unless otherwise specified). The parameter $\kappa = 2\pi H^{(2)}/l_1$ is introduced. The uniaxial tension and uniaxial compression are investigated separately.

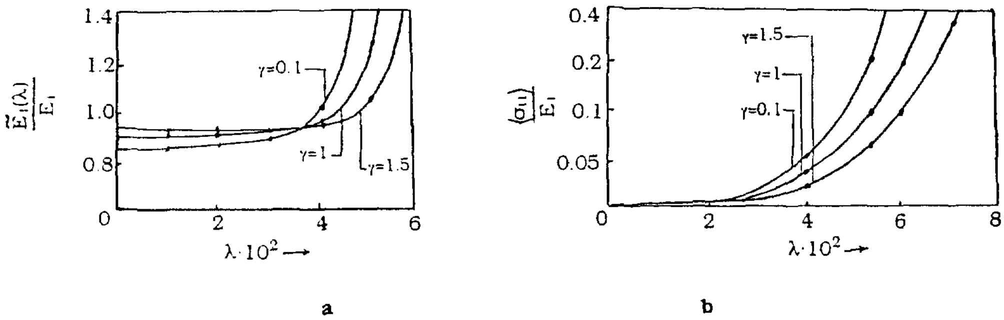

Fig. 3. Graphs of relations between $\tilde{E}_1(\lambda)/E_1$, $\langle\sigma_{11}\rangle/E_1$ and deformation λ at $\eta^{(2)} = 0.5$. In the case of tension.

Uniaxial tension

In this case, we suppose that $p_1 > 0$. The dependences between $\tilde{E}_1(\lambda)/E_1$ and λ (Fig. 3a) and between $\langle\sigma_{11}\rangle$ and λ (Fig. 3b) are shown in Fig. 3 for the cases $\gamma = 0.1$, 1.0, 1.5 at $E^{(2)}/E^{(1)} = 50$, $\eta^{(2)} = 0.5$, $\kappa = 0.1$. These graphs show that at $\lambda \leqslant 0.04$ with increasing λ the stiffness of the considered composite increases insignificantly. However, with $\lambda > 0.04$ with increasing λ the above-mentioned stiffness increases sharply. Furthermore, it follows from the graphs shown in Fig. 3b that in this case, for $\lambda \leqslant 0.03$ the dependences between $\langle\sigma_{11}\rangle/E_1$ and λ are linear with sufficiently high accuracy. For $\lambda > 0.03$ the above dependences are characterized by clearly defined nonlinearity. Note that analogous results are obtained with other values of parameters of the considered problem. Moreover, note that from comparison of the graphs constructed using various values of γ it follows that by increasing γ the degree of the nonlinearity of investigated dependences weakens insignificantly.

To investigate the influence of the change of $E^{(2)}/E^{(1)}$ on the character of the above dependences, consider the graphs in Fig. 4, which are constructed with the use of values $\eta^{(2)} = 0.5$, $\kappa = 0.05$, $\gamma = 0.1$. These graphs show the relations between $\tilde{E}_1(\lambda)/E_1$ and λ (Fig. 4a), and between $\langle\sigma_{11}\rangle/E_1$ and λ as well (Fig. 4b) with $E^{(2)}/E^{(1)} = 20$, 25, 30, 35, 50 and 100. It follows that with increasing $E^{(2)}/E^{(1)}$, the non-linearity of the investigated dependences begins in the early stage of the uniaxial deformation in the direction of Ox_1 axis from these results.

Consider the curves shown in Fig. 5 and constructed at $\gamma = 0.1$, 1.0, 1.5 which indicate the relations between $\tilde{E}_1(\lambda)/E_1$ and λ (Fig. 5a) and between $\langle\sigma_{11}\rangle/E_1$ and λ (Fig. 5b) as well in the case $\eta^{(2)} = 0.35$, $\kappa = 0.1$, $E^{(2)}/E^{(1)} = 50$. The comparison of these results with those given in Fig. 3, obtained with the same values of the parameters κ, $E^{(2)}/E^{(1)}$ and γ in the case $\eta^{(2)} = 0.5$, proves that by decreasing the filler concentration in the considered composite (i.e. $\eta^{(2)}$), the value of the deformation (i.e. λ) after which the nonlinearity of investigated dependences begins, increases.

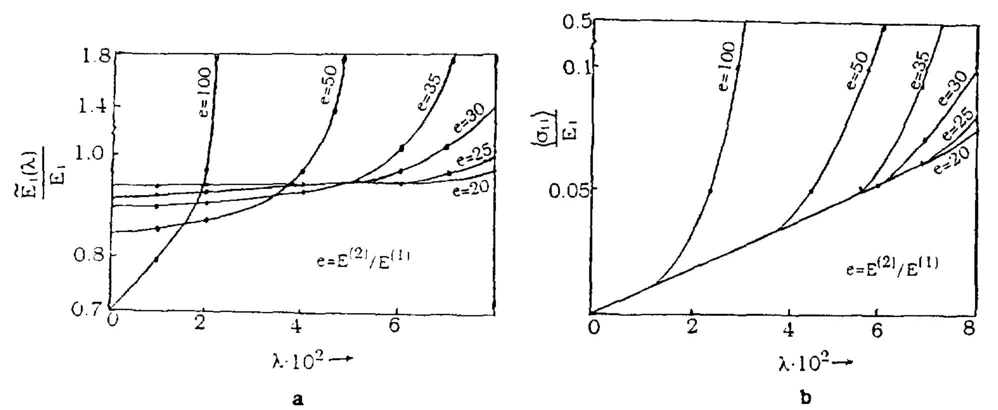

Fig. 4. Graphs illustrating the influence of the change of $E^{(2)}/E^{(1)}$ on the character of relations between $\tilde{E}_1(\lambda)/E_1$, $\langle\sigma_{11}\rangle/E_1$ and deformation λ. In the case of tension.

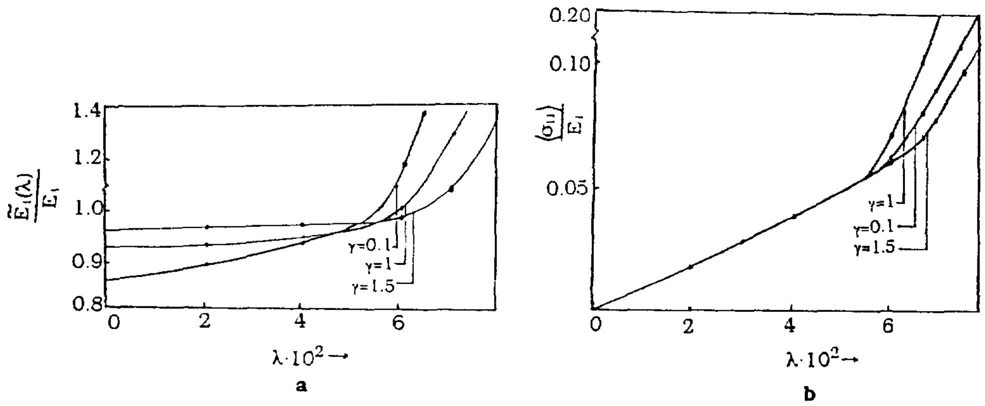

Fig. 5. Graphs of relations between $\tilde{E}_1(\lambda)/E_1$, $\langle\sigma_{11}\rangle/E_1$ and deformation λ at $\eta^{(2)} = 0.35$. In the case of tension.

With the above result we restrict ourselves to the analysis of the results obtained in uniaxial tension of the considered composite material in the direction of the Ox_1 axis (Fig. 1.) From this analysis we may conclude that by growing deformation, the curvature of reinforcing layers in the composite material leads to increase of the stiffness under the tension of the considered material.

Uniaxial compression

To investigate the relations between $\tilde{E}_1(\lambda)/E_1$ and λ and between $\langle\sigma_{11}\rangle/E_1$ and λ in the case when $p_1 < 0$. The graphs of the above relations are shown in Fig. 6 for the cases $\gamma = 0.1$, 1.0, 1.5 at $\eta^{(2)} = 0.5$, $E^{(2)}/E^{(1)} = 50$ and $\kappa = 0.1$, 0.02.

The above curves show that by increasing λ the values of $\tilde{E}_1(\lambda)$ (Fig. 6a) decrease steadily. Moreover, these curves show that subsequent to a certain stage of the deformation, the dependences between $\langle\sigma_{11}\rangle$ and λ (Fig. 6b) become nonlinear. Note that unlike the uniaxial tension in the considered case, i.e. in the uniaxial compresion, by increasing λ, the stiffness of the composite material with the structure considered, reduces.

The comparison of results obtained for various values of κ and γ shows that with increasing the pliancy (i.e. by decreasing the values of κ) of reinforcing layers, as well as, by decreasing γ, the above nonlinearity (between $\langle\sigma_{11}\rangle/E_1$ and λ) arises in an earlier stage of the compression deformation.

The graphs (Fig. 7) constructed with use of the values $\varepsilon = 0.015$, $\gamma = 0.1$, $\eta^{(2)} = 0.5$ and $\kappa = 0.1$ demonstrate the influence of the change of $E^{(2)}/E^{(1)}$ on the character of the investigated dependences. It follows that by increasing $E^{(2)}/E^{(1)}$ the nonlinear character of

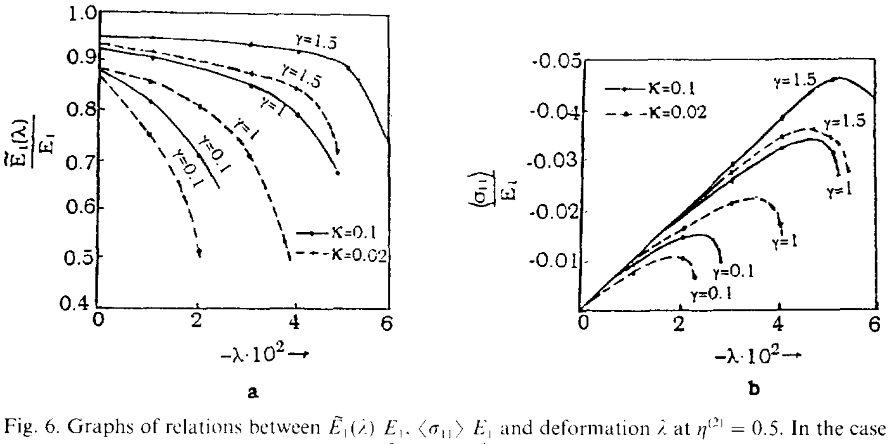

Fig. 6. Graphs of relations between $\tilde{E}_1(\lambda)/E_1$, $\langle\sigma_{11}\rangle/E_1$ and deformation λ at $\eta^{(2)} = 0.5$. In the case of compression.

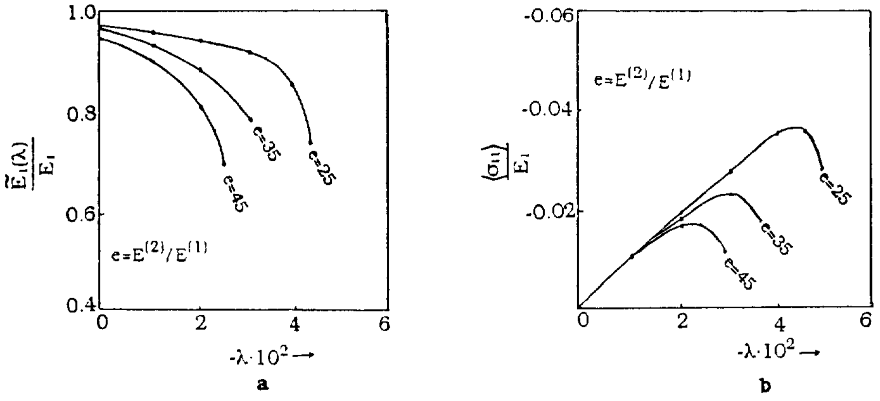

Fig. 7. Graphs illustrating the influence of the change of $E^{(2)}/E^{(1)}$ on the character of relations between $\tilde{E}_1(\lambda)/E_1$, $\langle\sigma_{11}\rangle/E_1$ and deformation λ. In the case of compression.

the dependences between $\tilde{E}_1(\lambda)/E_1$ and λ (Fig. 7a) and between $\langle\sigma_{11}\rangle/E_1$ and λ (Fig. 7b) is observed in an earlier stage of the deformation.

Note the following conditions obtained from the results shown in Figs 6 and 7. These results prove that in the relation between $\langle\sigma_{11}\rangle/E_1$ and λ there is a value of λ under which

$$\frac{d\langle\sigma_{11}\rangle}{d\lambda} = 0. \tag{45}$$

Subsequent to this value of λ, with an insignificant increase in λ, the stress $\langle\sigma_{11}\rangle$ decreases sharply. It is known that the analogous phenomenon in fracture mechanics is called "material instabilities", and in certain cases is applied as a criterion of fracture of materials. From the above we may conclude that the relation (45) can be applied as a criterion of fracture of the considered composite materials under uniaxial compresion in the direction of the Ox_1 axis (see Fig. 1). In the opinion of the author, the theoretical and experimental study of the fracture of the considered composite materials by applying the criterion (45) is the subject of important separate investigations.

Thus, taking into account the obtained results, we may conclude that under uniaxial compression of the considered composite in the direction of the Ox_1 axis (see Fig. 1) by increasing the deformation, the stiffness of this material decreases.

Considering the reliability of the analysed numerical results, which have been obtained with the use of only the first two terms of the series (40), in the sense of the numerical convergence of these results. Note that preliminary investigations of the stress-deformation state in the considered composite material in the framework of the above formulation, show that in the range of values of the structural parameters of the considered problem, the contribution of the third and subsequent orders of approximations on the values of the stress and deformations have the order of 10^{-5}–10^{-6}. Moreover, note that the index $2k$ in γ_{2k}, which enters expression (40), shows the number of the approximation order up to which all previous order approximations enter the expressions of γ_{2k}. For example, the values of the zeroth, the first and the second order approximations enter the expression of γ_2 (42). Consequently, it follows from the above and from the proposed approach that the contributions of $\gamma_4, \gamma_6, \ldots$ to the values of U (40) will also have the order of 10^{-5}–10^{-6}. Thus, the numerical results analysed in this paper may be considered as sufficiently reliable in the sense of numerical convergence.

It is obvious that in other cases which do not enter the framework of the above stated cases, it is necessary to take into account the subsequent order approximations (i.e. γ_4, $\gamma_6, \ldots$) for the calculation of the considered normalized nonlinear mechanical properties.

In conclusion, note that the method proposed in this paper, with obvious changes, may be applied for the other normalized nonlinear mechanical properties of composite materials with curved layers.

REFERENCES

Akbarov, S. D. and Guz', A. N. (1984). On the method of problems solution in mechanics of composite materials with curved layers. *Prikl. Mekh.* **20**(4), 3–9 (In Russian).

Akbarov, S. D. and Guz', A. N. (1985). On mechanics of composite material with curved structures. *Dokl. Akad. Nauk USSR* **281**, 37–44 (In Russian).

Akbarov, S. D. and Guz' A. N. (1992) Statics of laminated and fibrous composites with curved structures. *Appl. Mech. Rev.* (published by the American Society of Mechanical Engineers) **45**, 17–35.

Akbarov, S. D., Movsumov, E. A. and Guliev, M. S. (1992). Definition of reduced mechanical properties of composite materials with periodically curved layers. *Prikl. Mekh.* **28**(10), 35–43. (In Russian)

Bazhant, Z. P. (1968) The influence of the curvature of reinforced fibres on the elasticity modulus and strength of composite materials. *Mechanika Polimerov* **2**, 314–321 (In Russian).

Bolotin, V. V. and Novichkov, Yu. N. (1980) *Mechanics of Multilayered Structures*. Mashinostroyeniye, Moscow (In Russian).

Khoroshun, L. P. and Maslov, B. P. (1980) *The Methods of Automatic Computation of Physico-Mechanical Constants of Composite Materials*. Naukova Dumka, Kiev (In Russian).

Whitney, J. M. (1966). Geometrical effects of filament twist on the modulus and strength of graphite fiber. *Reinforced Composites Textil. Res. J.* **36**, 765–770.